

Connection between the high energy-scale evolution of the P- and T-odd πNN coupling constant and the strong πNN interaction

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Abstract

The large energy-scale behaviour of the parity and time-reversal violating (PTV) pion-nucleon coupling constant is analyzed in a model combining renormalization-group techniques and the dressing of the PTV vertex with a pion loop. With the strong πNN vertex as a mixture of the pseudovector and pseudoscalar couplings, we show that depending on the admixture parameter, two qualitatively distinct types of behaviour are obtained for the PTV coupling constant at high energy scales: an asymptotic freedom or a fixed-point. We find a critical value of the admixture parameter which delineates these two scenarios. Several examples of the high-energy scale behaviour of the PTV πNN constant are considered, corresponding to realistic hadronic models of the strong pion-nucleon interaction.

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In addition to the usual strong interaction, pions and nucleons may have a much suppressed Parity- and Time-reversal Violating (PTV) interaction proportional to the Charge conjugation and Parity (CP) violating θ term in the QCD Lagrangian [1]. The upper bound for the PTV πNN coupling constant, denoted here as c , can be extracted from measurements of Electric Dipole Moments (EDMs) of the neutron and various atomic and molecular systems. It is found to be extremely small: $c < 7 \times 10^{-12}$, corresponding to $|\theta| < 3 \times 10^{-10}$ [1, 2]. At higher energies, the PTV effects may either remain suppressed, or alternatively they may grow leading to a significant CP violation. Which of these two scenarios is more likely to occur has important implications for possible extensions of the Standard Model.

In a previous paper [3] a hadronic model was developed in which a PTV πNN vertex was dressed with meson loops. Using a renormalization-group approach, it was found that if one uses a purely pseudoscalar strong πNN vertex in the loop, the PTV coupling vanishes at high energy scales (an asymptotic freedom). Alternatively, by using a purely pseudovector strong πNN vertex, one obtains a PTV coupling constant tending to a fixed value at high energies. In this report we generalize these results to the case where a more realistic strong πNN vertex is used in the loop integral for the PTV πNN coupling.

To obtain a good description of nucleon-nucleon and pion-nucleon scattering, as well as pion photoproduction and other processes up to resonance energies, one often uses a strong πNN vertex as a mixture of the pseudovector (PV) and pseudoscalar (PS) interactions:

$$\Gamma_S = g \left\{ \frac{a}{2m} \gamma_5 \not{q} + (1 - a) \gamma_5 \right\}, \quad (1)$$

where q is the pion four-momentum and $g \approx 13$ is the strong pion-nucleon coupling constant. In different approaches, the PV-PS mixing parameter a appears either as a constant [4, 5, 6, 7] fitted to data, or as an energy-dependent function which is determined from a phenomenological fit, as in [8], or obtained from a self-consistent dressing of the vertex, as in [9].

The energy-scale evolution of the PTV coupling constant can be studied using renormalization-group techniques. The renormalization-group equation for the dependence of the coupling constant c on the energy scale μ reads [10]

$$\mu \frac{dc}{d\mu} = \beta[c], \quad (2)$$

where $\beta[c]$ is the beta-function. In this report we restrict ourselves to the system of nucleons and pions only while generalizing the calculation of Ref. [3] to a more general strong πNN

vertex given in Eq. (1), with an arbitrary parameter a . Following the approach of [3], we extract the beta-function from the one-loop diagrams depicted in Fig. (1), within the standard procedures [10] of dimensional regularization and modified minimal subtraction. The resulting beta-function can be written

$$\beta[c] = Ac + Bc^3, \quad (3)$$

where

$$A = (5a^2 - 2)\frac{g^2}{16\pi^2}, \quad B = -\frac{1}{8\pi^2}. \quad (4)$$

The renormalization-group equation (2) can be integrated between energy scales μ_1 and μ_2 , yielding the solution

$$\left(\frac{\mu_2}{\mu_1}\right)^A = \frac{c(\mu_2)}{c(\mu_1)} \sqrt{\frac{1 + \frac{B}{A}c^2(\mu_1)}{1 + \frac{B}{A}c^2(\mu_2)}}. \quad (5)$$

Approximations to this solution for smaller and larger values of $c(\mu)$ have simpler forms, as was discussed in some detail in Ref. [3].

It is well-known [10] that, through the renormalization-group equation (2), the sign of the beta-function determines the behaviour of the PTV coupling constant $c(\mu)$ at large energy scales μ . Two qualitatively distinct types of behaviour were identified in Ref. [3]. In the first type $\beta[c] < 0$, $\forall c > 0$, entailing $c(\mu \rightarrow \infty) \rightarrow 0$, i. e. an asymptotic freedom behaviour. This scenario obtains if one chooses $a = 0$ in Eq. (1), thus using a purely pseudoscalar strong πNN vertex. If one uses a purely pseudovector vertex, setting $a = 1$ in Eq. (1), the second scenario is realized. In this case the beta-function is positive for moderate values of c , in which interval $\beta[c]$ increases, reaches a maximum value and then decreases, becoming negative after crossing zero at a fixed point $c^* \neq 0$. This behaviour leads to $c(\mu \rightarrow \infty) \rightarrow c^*$, i. e. the PTV coupling constant approaches the fixed point c^* at sufficiently high energies. Since many realistic hadronic models of strong pion-nucleon interaction at low and intermediate energies use a mixture of the pseudovector and pseudoscalar vertices, a question arises what kind of high-energy behaviour of the PTV coupling constant corresponds to these hadronic models. We study this question in the remainder of this report.

Assuming without loss of generality that $c > 0$, the beta-function in Eq. (3) will be negative-definite if the PV-PS ratio $a \leq a_{crit}$, where the critical value is equal to

$$a_{crit} = \sqrt{\frac{2}{5}} \approx 0.63. \quad (6)$$

This critical value allows us to establish a connection between hadronic models with a strong πNN interaction and the asymptotic behaviour of the PTV πNN coupling constant. Specifically, a model with $a \leq a_{crit}$ is consistent with the asymptotic freedom behaviour of the PTV coupling constant, $c(\mu \rightarrow \infty) \rightarrow 0$, whereas a model with $a > a_{crit}$ advocates the fixed point scenario, $c(\mu \rightarrow \infty) \rightarrow c^*$. The fixed value c^* is determined by solving the equation $\beta[c^*] = 0$. Using Eq. (3), we obtain

$$c^* = g \sqrt{\frac{5a^2 - 2}{2}}. \quad (7)$$

The two types of high-energy behaviour described above coincide when $c^* \rightarrow 0$, which happens for $a \rightarrow a_{crit}$.

The accompanying table is a compilation of results of several successful hadronic models [4, 5, 6, 7] in which the mixing parameter a in the strong πNN vertex Eq. (1) is assumed to be constant. From the relation between a and a_{crit} of Eq. (6) we obtain the high-energy limit of the PTV pion-nucleon coupling constant c , given in the last column of the table (Eq. (7), with $g \approx 13$, is used to determine the values of c^*).

Model	$\approx a$	$c(\mu \rightarrow \infty)$
[4] Relativistic	0.78	$c^* \approx 9.4$
[4] Non-relativistic	0.59	0
[5]	0.75	$c^* \approx 8.3$
[6]	0.97	$c^* \approx 15.1$
[7]	0.8	$c^* \approx 10.1$

The hadronic models in which the mixing parameter a is a function of energy are exemplified by Refs. [8] and [9]. Although the details of these two models are different, they both describe a strong πNN interaction which is essentially pseudovector at zero energy, but with a pseudoscalar component increasing with energy. This translates to a mixing parameter a being unity at zero energy and decreasing monotonously at higher energies. If at sufficiently high energies a becomes smaller than a_{crit} , then $c(\mu \rightarrow \infty) \rightarrow 0$. Generally it is not known whether the models [8] and [9] are applicable at energies sufficiently high for the renormalization-group considerations. Therefore the asymptotic behaviour $c(\mu \rightarrow \infty)$ corresponding to these models cannot be determined unambiguously.

To summarize, in this report we have linked the PV-PS mixture in a strong πNN vertex with the type of the high-energy scale evolution of the PTV πNN coupling constant c .

Using the one-loop beta-function, we have found a critical values of the PV-PS admixture parameter a , which separates the scenarios of asymptotic freedom and a fixed point for $c(\mu \rightarrow \infty)$. For illustration, we have calculated the fixed points for several hadronic models. At present, we restricted our model to the nucleon and pion degrees of freedom only, which made possible the direct connection with the hadronic models of the strong πNN interaction. This approach, however, is less reliable at sufficiently high energies, where heavier mesons, as well as quark-gluon degrees of freedom, are expected to become important.

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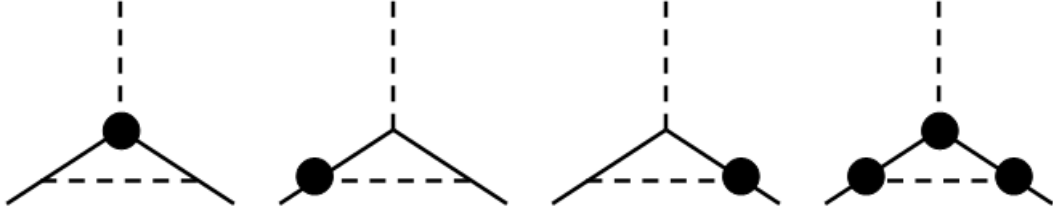


FIG. 1: Loop diagrams used to extract the beta-function. The solid and dashed lines are nucleons and pions. The blob represents a PTV πNN vertex described by the coupling constant c . The point vertex represents the strong πNN vertex given in Eq. (1).

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